Summary of the PhD thesis
Representative coordinates and the geometry of the Bergman metric

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The representative coordinates of the domain $\Omega \subset\subset \mathbb{C}^n$, at the point $z_0 \in \Omega$ $i = 1, ..., n$, are defined as follows

$$w_i(z) = \sum_{j=1}^{n} T^{\bar{j}i}(z_0) \frac{\partial}{\partial \bar{\zeta}_j} \log \frac{K(z,\zeta)}{K(\zeta,\zeta)}_{|\zeta = z_0},$$

$i = 1, ..., n$, where $T^{\bar{j}i}(z_0)$ is the inverse matrix of $(\frac{\partial^2}{\partial z_i \partial \bar{z}_j} \log K(z,z))_{i,j=1..n}$ and $K(z,\zeta)$ is the Bergman kernel of the domain Ω. It is known that the functions w_i give a local coordinates of some neighbourhood of the point z_0, however how exactly this neighbourhood looks like remains unknown. In this PhD thesis we prove that it is possible to obtain control over the radius of the maximal geodesic ball with respect to the Bergman metric, with center at z_0, in which the mapping $w : \Omega \ni z \rightarrow (w_1(z), w_2(z), ..., w_n(z))^t \in \mathbb{C}^n$ is well defined and regular i.e., it's jacobian does not vanish. The examples, provided in the last section show that the obtained estimates are optimal in the general setting.

A variation of the aforementioned result, valid on complex manifolds is also provided, with the goal of explaining the differences between the $\mathbb{C}^n$ and the manifold settings in mind.

The second result is the example of a planar domain for which the curvature of the Bergman metric behaves in an unexpected way, achieving all possible (formerly theoretically predicted) value, that is $(-\infty, 2)$. It is somehow paradoxical that such examples for different curvatures (holomorphic sectional curvature, Ricci curvature, Scalar curvature) of the Bergman metric were known before only for dimensions greater than 1.

A common aspect of the both results is the study of the Ricci curvature of the Bergman metric, which actually appears in the aforementioned estimates.